Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 10

Vectors

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate’s grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$.

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D. **(2)**

$b - a = d - b$

$2b - a = d$

$2\begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} = \begin{pmatrix}8\\-4\\6\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} = \begin{pmatrix}6\\-7\\10\end{pmatrix}$

$\therefore d = \begin{pmatrix}6\\-7\\10\end{pmatrix}$

Given $|\overrightarrow{AC}| = 4$,

(b) find the value of a. **(3)**

$|C - A| = 4$

$|(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) - (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})| = 4$

$|(a-2)\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}| = 4 \quad = \sqrt{(a-2)^2 + 4 + 4} = \sqrt{a^2 - 4a + 12}$

$16 = a^2 - 4a + 12$

$0 = a^2 - 4a - 4.$

$a = 2 - 2\sqrt{2}$ OR $2 + 2\sqrt{2}$

not possible as $a < 0$

(Total for Question 1 is 5 marks)

2. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(*a*) Find $\overrightarrow{AB}$

$B - A$. $\begin{pmatrix} 3i \\ -3j \\ -4k \end{pmatrix} - \begin{pmatrix} 2i \\ 5j \\ -6k \end{pmatrix} = \begin{pmatrix} i \\ -8j \\ 2k \end{pmatrix}$

(2)

(*b*) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

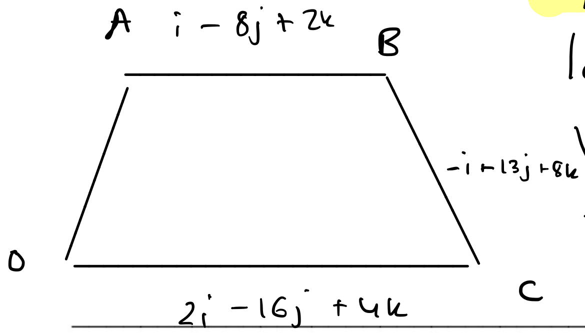
(2)

$\overrightarrow{BC} = c - b = \begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ -13 \\ 8 \end{pmatrix}$

$\overrightarrow{OC} = 2i - 16j + 4k$. $\rightarrow$ $\overrightarrow{OC}$ and $\overrightarrow{BC}$ are not multiples so not parallel

and $\overrightarrow{OC}$ is $n(\overrightarrow{AB})$, therefore $\overrightarrow{OC}$ and $\overrightarrow{AB}$ are parallel

Are $\overrightarrow{AB}$ and $\overrightarrow{OC}$ equal lengths?



$|\overrightarrow{OC}| = \sqrt{2^2 + 16^2 + 4^2} = 2\sqrt{69}$

$|\overrightarrow{AB}| = \sqrt{1^2 + 8^2 + 2^2} = \sqrt{69}$

$\therefore$ not equal lengths so must be a trapezium

(Total for Question 2 is 4 marks)

3. Relative to a fixed origin O, the points A and B are such that

$$\overrightarrow{OA} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}, \text{ where } p \text{ is a constant,}$$

and the points C and D are such that

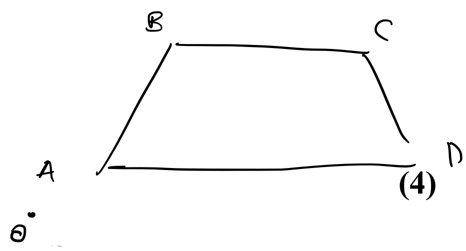
$$\overrightarrow{BC} = \begin{pmatrix} 0 \\ 6 \\ -7 \end{pmatrix} \text{ and } \overrightarrow{AD} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}.$$

(a) Find the position vector of the point D.

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix}$$

(1)

Given that $ABCD$ is a trapezium,



(b) find the value of p.

(4)

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} + \begin{pmatrix} 0 \\ 6 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ p-7 \end{pmatrix}$$

$$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$$

$$= \begin{pmatrix} 3 \\ 5 \\ p-7 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ p-10 \end{pmatrix}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ p-7 \end{pmatrix}$$

$$\overrightarrow{AB} = \frac{3}{2}\overrightarrow{DC}$$

$\therefore$ for z

$$p - 7 = \frac{3}{2}(p - 10)$$

$$p - 7 = \frac{3}{2}p - 15 \rightarrow 8 = \frac{1}{2}p \rightarrow \therefore p = 16$$

(Total for Question 3 is 5 marks)

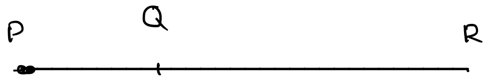
4. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively. Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

(3)

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$



O

let $\overrightarrow{OP} = p \qquad \overrightarrow{OQ} = q$

$\overrightarrow{OR} = r$

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -p + r$

$\rightarrow \overrightarrow{PQ} = \frac{1}{3}\overrightarrow{PR} = -\frac{1}{3}p + \frac{1}{3}r$

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -p + q$

$\therefore \quad -\frac{1}{3}p + \frac{1}{3}r = -p + q$

$\frac{2}{3}p + \frac{1}{3}r = q$

$\frac{1}{3}(2p + r) = q \quad =$ as required.

(Total for Question 4 is 3 marks)

5.

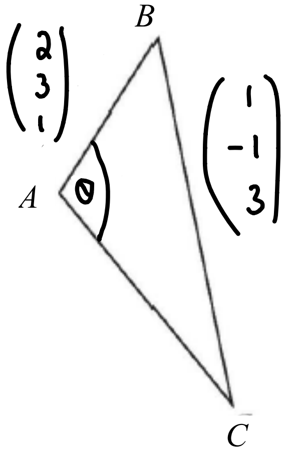


Figure 2

Figure 2 Figure 2 shows a sketch of a triangle ABC.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

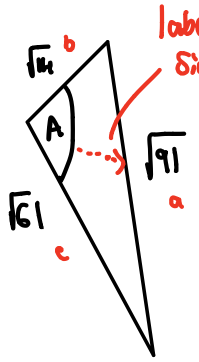
show that $\angle BAC = 105.9°$ to one decimal place.

(5)

$|AB| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$ and $|BC| = \sqrt{1^2 + (-9)^2 + 3^2} = \sqrt{91}$

$\Rightarrow AC = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -9 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 4 \end{pmatrix}$ then $|AC| = \sqrt{3^2 + (-6)^2 + 4^2} = \sqrt{61}$

$\Rightarrow$



label a as the opposite side to the angle A.

$\Rightarrow$ Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$91 = 14 + 61 - 2\sqrt{14}\sqrt{61}\cos A$

$16 = -2\sqrt{854}\cos A$

$A = \cos^{-1}\left(-\dfrac{16}{2\sqrt{854}}\right) = 105.887...$

$\Rightarrow \underline{\angle BAC = 105.9°}$ as required

(Total for Question 5 is 5 marks)

6. Relative to a fixed origin O,

- the point A has position vector $-2\mathbf{i} + 3\mathbf{j}$,
- the point B has position vector $3\mathbf{i} + p\mathbf{j}$, where p is constant,
- the point C has position vector $q\mathbf{i} + 7\mathbf{j}$, where q is constant.

Given that $|\overrightarrow{AB}| = 5\sqrt{2}$,

(a) find the possible values for p.

(3)

$|\overrightarrow{AB}| = 5\sqrt{2}$

$|b - a| = 5\sqrt{2}$

$|(3i + pj) - (-2i + 3j)| = 5\sqrt{2}$

$|5i + (p-3)j| = 5\sqrt{2}$

$\sqrt{5^2 i + (p-3)^2} = 5\sqrt{2} \rightarrow 25 + (p^2 - 6p + 9) = 50$

$p^2 - 6p - 16 = 0$

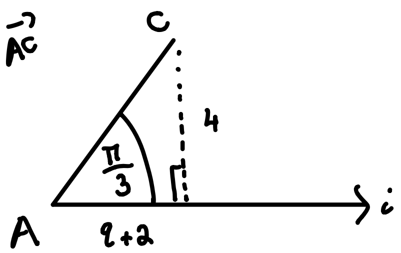
$(p - 8)(p + 2) = 0$

$p = 8$ OR $p = -2$

Given that the angle between $\overrightarrow{AC}$ and the unit vector $\mathbf{i}$ is $\frac{\pi}{3}$ radians,

(b) find the exact value of q.

(3)



$\overrightarrow{AC} = c - a = \begin{pmatrix} q \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} q+2 \\ 4 \end{pmatrix}$ – i component, – j component

$\tan\theta = \frac{o}{a} \Rightarrow \tan\left(\frac{\pi}{3}\right) = \frac{4}{q+2}$

$\sqrt{3}(q+2) = 4 \Rightarrow \sqrt{3}q + 2\sqrt{3} = 4$

$q = \frac{4 - 2\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3} - 6}{3}$

$\Rightarrow q = \frac{4\sqrt{3} - 6}{3}$

(Total for Question 6 is 6 marks)

7.

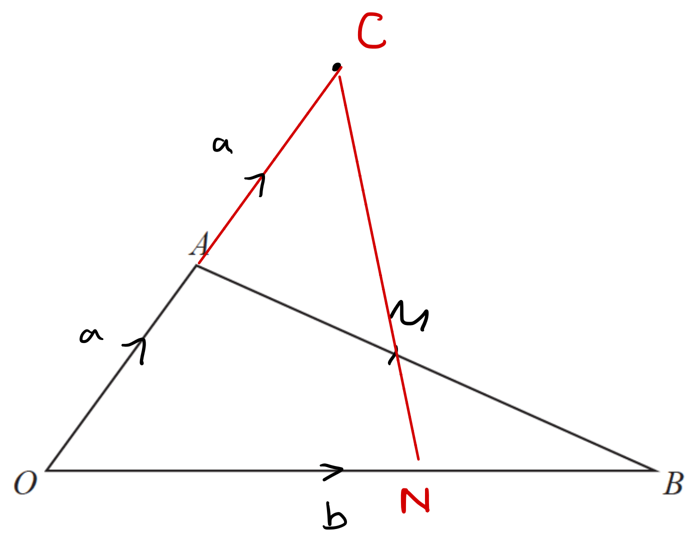


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(*a*) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **(2)**

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\overrightarrow{AM} = \frac{1}{2}(\underline{b} - \underline{a})$

$\overrightarrow{CA} = -\underline{a}$

$\therefore \overrightarrow{CM}$:

$= -\underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= -\underline{a} - \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

$= -\frac{3}{2}\underline{a} + \frac{1}{2}\underline{b}$

(*b*) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant. **(2)**

$\overrightarrow{CM} = \lambda\overrightarrow{CN} \longrightarrow \overrightarrow{MN} = (1-\lambda)\overrightarrow{CN}$

$\overrightarrow{ON} = n\underline{b} \longrightarrow \overrightarrow{NB} = (1-n)\underline{b}$

$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = 2\underline{a} + \frac{1}{\lambda}\overrightarrow{CM} = 2a + \frac{1}{\lambda}\left(-\frac{3}{2}a + \frac{1}{2}b\right)$

$\overrightarrow{ON} = \left(2 - \frac{3}{2\lambda}\right)a + \frac{1}{2\lambda}b \longrightarrow$ as required

$\underline{a}$ component $= 0$

$2 - \frac{3}{2}\lambda = 0$

$2 = \frac{3}{2\lambda}$

$4\lambda = 3$

$\lambda = \frac{3}{4}$

$\therefore \overrightarrow{ON} = \left(2 - \frac{3}{2\left(\frac{3}{4}\right)}\right)a + \frac{1}{2\left(\frac{3}{4}\right)}b$

$\overrightarrow{ON} = \frac{4}{6}b$

(*c*) Hence prove that $ON : NB = 2 : 1$

(2)

$\overrightarrow{ON} = \frac{4}{6}\underline{b} = \frac{2}{3}\underline{b}$

$\overrightarrow{NB} = \left(1 - \frac{2}{3}\right)\underline{b} = \frac{1}{3}\underline{b}$

$\frac{2}{3}b : \frac{1}{3}b \longrightarrow \underline{\underline{2 : 1}}$ as required

(Total for Question 7 is 6 marks)